Pontrjagin duality on multiplicative gerbes

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Contents

Quantum groups

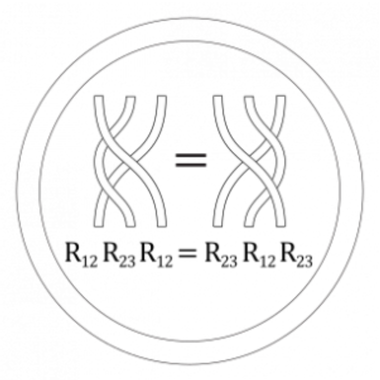
They could be thought as "deformations" of classical groups.

They should be taken as the

"Symmetry groups"

Include: Hopf algebras and their generalizations.

Yang-Baxter equation



Condition for integrability of 1D and 2D quantum systems.

Quantum integrable group –> Quasi-triangular Hopf Algebra

Drinfeld Double

H finite dimensional Hopf $\mathbb{C}$-algebra.

$H^* = \mathrm{Hom}_{\mathbb{C}}(\mathrm{H}, \mathbb{C})$, dual Hopf algebra.

$D(H) := H \otimes H^*$ (Drinfeld Double)

The image of the identity map

$$\mathrm{Hom}_{\mathbb{C}}(\mathrm{H}, \mathrm{H}) \cong \mathrm{H}^* \otimes \mathrm{H}$$

provides an R-matrix that satisfies the Yang-Baxter equation. If $\{v_i\}_{i \in I}$ is a basis for H, and $\{v^i\}_{i \in I}$ is the dual basis for H^*:

$$\boxed{R = \sum_{i \in I} (1 \otimes v^i) \otimes (v_i \otimes 1) \in \mathrm{End}(\mathrm{D}(\mathrm{H}) \otimes \mathrm{D}(\mathrm{H}))}$$

Drinfeld Double of finite group $D(G)$

$D(G) = \mathbb{C}G \otimes \mathbb{C}^G$. Base: $\{g \otimes \delta_x | (g, x) \in G\}$.

$$(g \otimes \delta_x)(h \otimes \delta_y) = \delta_{x,hyh^{-1}}(gh \otimes \delta_y), \qquad 1 = \sum_{g \in G} 1 \otimes \delta_x$$

$$\Delta(g \otimes \delta_x) = \sum_{ab=x} (g \otimes \delta_a) \otimes (g \otimes \delta_b), \qquad \epsilon(g \otimes \delta_x) = \delta_{x,1}$$

$$S(g \otimes \delta_x) = g^{-1} \otimes \delta_{g^{-1}x^{-1}g}$$

$$R = \sum_{x,y \in G} (1 \otimes \delta_x) \otimes (x \otimes \delta_y)$$

Twisted Drinfeld Double of finite group $D^\omega(G)$

$\omega \in Z^3(G, U(1))$ normalized cocycle. Define:

$$\beta_x(g,h) = \frac{\omega(g,h,x)\omega(ghxh^{-1}g^{-}1,g,h)}{\omega(g,hxh^{-1},h)}$$

$$\mu_g(x,y) = \frac{\omega(gxg^{-}1,gyg^{-}1,g)\omega(g,x,y)}{\omega(gxg^{-1},g,y)}$$

$$(g \otimes \delta_x)(h \otimes \delta_y) = \delta_{x,hyh^{-1}}\beta_y(g,h)(gh \otimes \delta_y), \qquad 1 = \sum_{g \in G} e \otimes \delta_x$$

$$\Delta(g \otimes \delta_x) = \sum_{ab=x} \mu_g(a,b)(g \otimes \delta_a) \otimes (g \otimes \delta_b), \qquad \epsilon(g \otimes \delta_x) = \delta_{x,1}$$

Twisted Drinfeld Double of finite group $D^\omega(G)$

$$S(g \otimes \delta_x) = \frac{1}{\beta_{x^{-1}}(g^{-1}, g)\mu_g(x, x^{-1})}(g^{-1} \otimes \delta_{g^{-1}x^{-1}g})$$

$$R = \sum_{x,y \in G} (1 \otimes \delta_x) \otimes (x \otimes \delta_y)$$

$$\Phi = \sum_{x,y,z \in G} \omega(x, y, z)^{-1}(1 \otimes \delta_x) \otimes (1 \otimes \delta_y) \otimes (1 \otimes \delta_z)$$

Note: (ω, β, μ) define a cocycle in $Z^3(G \ltimes G, U(1))$ via the pullback of the multiplication map

$$G \ltimes G \to G, \quad (g, x) \mapsto gx$$

Representations of $D^\omega(G)$

Category of representations $Rep(D^\omega(G))$ of the Twisted Drinfeld Double $D^\omega(G)$ becomes a Modular Tensor Category.

Modular Tensor Category: non-degenerate braided fusion category with a choice of spherical structure.

Extended Chern-Simons Field theory (Reshetikhin-Turaev, Freed):

$$F : \mathrm{Bord}_{\langle 3,2,1\rangle} \to \mathrm{Cat}_{\mathbb{C}}$$
$$F(S^1) \mapsto Rep(D^\omega(G))$$

Twisted equivariant K-theory

The isomorphism classes of representations

$$\pi_0(Rep(D^\omega(G)))$$

could be interpreted as the G-Twisted equivariant K-theory groups of G (Freed-Hopkins-Teleman, Willerton,...)

$$\boxed{{}^{\tau(\omega)}K_G(G) \cong \bigoplus_{(x)\in Conj(G)} {}^{\beta_x}R(C_G(x))}$$

with "pull-push" product structure.

Pointed fusion category $\mathrm{Vec}^{\omega}(\mathrm{G})$

The finite tensor category $\mathrm{Vec}^{\omega}(\mathrm{G})$ is a *pointed fusion category*, i.e. all its simple objects are invertible.

Objects: G-graded vector spaces $V = \oplus_{g \in G} V_g$.

Monoidal structure: $(V \otimes W)_k = \bigoplus_{hg=k} V_h \otimes W_g$.

Associativity constraint: $(V_h \otimes W_g) \otimes Z_k \xrightarrow{\omega(h,g,k)} V_h \otimes (W_g \otimes Z_k)$

2-Category of module categories over $\mathrm{Vec}^{\omega}(\mathrm{G})$

A module category $\mathcal{M}$ over the tensor category $\mathcal{C}$ consist of a functor

$$\mathcal{M} \otimes \mathcal{C} \to \mathcal{M}$$

and functorial associativity

$$\mu_{M,X,Y} : M \otimes (X \otimes Y) \xrightarrow{\sim} (M \otimes X) \otimes Y$$

satisfying the pentagon axiom, + more structures.

Idecomposable modules: $\mathcal{M}(A \backslash G, \mu)$ with $\delta_G \mu = \pi^* \omega$.

Dual category of $\mathcal{C} = \mathrm{Vec}^{\omega}(\mathrm{G})$

For $\mathcal{M}$ an idecomposable module category over $\mathrm{Vec}^{\omega}(\mathrm{G})$, the dual category is:

$$\boxed{\mathcal{C}^*_{\mathcal{M}} = \mathrm{End}_{\mathcal{C}}(\mathcal{M})}$$

For $\mathcal{C}$ and $\mathcal{M}$ semisimple, $\mathcal{C}^*_{\mathcal{M}}$ is semisimple and

$$\boxed{(\mathcal{C}^*_{\mathcal{M}})^*_{\mathcal{M}} \cong \mathcal{C}}$$

(Drinfeld) Center of monoidal category

Consider $\mathcal{C}$ as a module category over $\mathcal{C} \otimes \mathcal{C}^{op}$. The center of $\mathcal{C}$ is:

$$\boxed{\mathcal{Z}(\mathcal{C}) = \mathrm{End}_{\mathcal{C}\otimes\mathcal{C}^{\mathrm{op}}}(\mathcal{C})}$$

For $\mathcal{C}$ a fusion category, its center becomes a Braided Fusion Category. Moreover,

$$\boxed{\mathcal{Z}(\mathrm{Vec}^{\omega}(\mathrm{G})) \cong \mathrm{Rep}(\mathrm{D}^{\omega}(\mathrm{G}))}$$

as Braided Fusion Categories.

Morita equivalence

Two fusion categories $\mathcal{C}$ and $\mathcal{D}$ are called *Morita equivalent* if there exists an idecomposable module category over $\mathcal{C}$ such that

$$\boxed{\mathcal{C}^*_{\mathcal{M}} \cong \mathcal{D}}$$

Morita equivalent fusion categories $\mathcal{C}$ and $\mathcal{D}$ have equivalent categories of module categories. Moreover

$$\boxed{\mathcal{Z}(\mathcal{C}) \cong \mathcal{Z}(\mathcal{D})}$$

Theorem (Etingof-Nikshish-Ostrik)
Fusion categories $\mathcal{C}$ and $\mathcal{D}$ are Morita equivalent if and only if $\mathcal{Z}(\mathcal{C})$ and $\mathcal{Z}(\mathcal{D})$ are equivalent as Braided Fusion Categories.

Since $Rep(D^\omega(G)) \cong \mathcal{Z}(\mathrm{Vec}^\omega(G))$

Therefore:

$$\boxed{\mathrm{Vec}^\omega(G) \simeq_{\mathrm{Morita}} \mathrm{Vec}^{\widetilde{\omega}}(\widetilde{G})} \leftrightarrow \boxed{\mathrm{Rep}(D^\omega(G)) \cong \mathrm{Rep}(D^{\widetilde{\omega}}(\widetilde{G}))}$$

Morita equivalent pointed fusion categories determine equivalent Chern Simons Extended Field theories.

So, when are $\mathrm{Vec}^\omega(G)$ and $\mathrm{Vec}^{\widehat{\omega}}(\widehat{G})$ Morita equivalent?

$\mathrm{Vec}^{\omega}(\mathrm{G}) \simeq_{\mathrm{Morita}} \mathrm{Vec}^{\widehat{\omega}}(\widehat{\mathrm{G}})$

Theorem [Uribe 2016]:

$\mathrm{Vec}^{\omega}(\mathrm{G})$ and $\mathrm{Vec}^{\widehat{\omega}}(\widehat{\mathrm{G}})$ are Morita equivalent, if and only if: -

$\boxed{\phi : G \cong A \rtimes_F K}$ with A abelian and $F \in Z^2(K, A)$.

- $\boxed{\widehat{\phi} : \widehat{G} \cong K \ltimes_{\widehat{F}} \widehat{A}}$ with $\widehat{A} = \mathrm{Hom}(\mathrm{A}, \mathrm{U}(1))$ and $\widehat{F} \in Z^2(K, \widehat{A})$.

- There exists $\epsilon : K^3 \to U(1)$ such that $\boxed{\delta_K \epsilon = \widehat{F} \wedge F}$

- $[\phi^* \eta] = [\omega]$ and $[\widehat{\phi}^* \widehat{\eta}] = [\widehat{\omega}]$ with

$$\eta((a_1, k_1), (a_2, k_2), (a_3, k_3)) := \widehat{F}(k_1, k_2)(a_3)\, \epsilon(k_1, k_2, k_3)$$
$$\widehat{\eta}((k_1, \rho_1), (k_2, \rho_2), (k_3, \rho_3)) := \epsilon(k_1, k_2, k_3)\, \rho_1(F(k_2, k_3))$$

Reminiscent of T-duality

$\boxed{\mathrm{Vec}^{\omega}(\mathrm{G})}$ $\boxed{\mathrm{Vec}^{\widehat{\omega}}(\widehat{\mathrm{G}})}$

$$\begin{array}{ccccccc}
A & & & A \times \widehat{A} & & & \widehat{A} \\
& \searrow & \swarrow & & \searrow & \swarrow & \\
& G & & & & \widehat{G} & \\
& & \overset{F}{\searrow} & & \overset{\widehat{F}}{\swarrow} & & \\
& & & K & & &
\end{array}$$

$$\omega = \widehat{F} \cdot \epsilon \qquad\qquad \widehat{\omega} = F \cdot \epsilon$$

Morita equivalence classes for $|G| = 4$

$\mathbb{Z}/2 \times \mathbb{Z}/2$	$\mathbb{Z}/4$
$\{0\}$	
$orb(x^4)$	
$orb(x^4 + x^2y^2 + y^4)$	
$orb(x^2y^2)$	$\{0\}$
	$\{u^2\}$
	$\{2u^2\}$
	$\{3u^2\}$

$$\mathrm{Vec}^{x^2y^2}(\mathbb{Z}/2 \times \mathbb{Z}/2) \simeq_{\mathrm{Morita}} \mathrm{Vec}(\mathbb{Z}/4)$$

Morita equivalence classes for $|G| = 8$

Together with student Munõz.

$(\mathbb{Z}/2)^3$	$\mathbb{Z}/4 \times \mathbb{Z}/2$	$\mathbb{Z}/8$	D_8	Q_8
$orb(x^2y^2)$	$\{0\}$			
$orb(x^4+y^2z^2)$	$\{v^2\}$			
$orb(x^2yz+xy^2z+xyz^2)$			$\{0\}$	
$orb(x^4+x^2yz+xy^2z+xyz^2)$			$\{\alpha+\beta,\beta\}$	$\{0\}$
$orb(x^4+x^2yz+xy^2z+xyz^2+y^2z^2)$			$\{\alpha\}$	
$orb(x^2yz+xy^2z+xyz^2+x^2y^2+x^2z^2+y^2z^2)$				$\{4t\}$
	$orb(uv)$	$\{0\}$		
	$orb(uv+u^2)$	$\{4s^2\}.$		

Case $|G| = p^3$ was done together with Maya and Mejia.

Isomorphism of Twisted Drinfeld Doubles

For $G = A \rtimes_F K$ and $\widehat{G} = K \ltimes_{\widehat{F}} \widehat{A}$ with

$$\omega((a_1, k_1), (a_2, k_2), (a_3, k_3)) := \widehat{F}(k_1, k_2)(a_3)\, \epsilon(k_1, k_2, k_3)$$
$$\widehat{\omega}((k_1, \rho_1), (k_2, \rho_2), (k_3, \rho_3)) := \epsilon(k_1, k_2, k_3)\, \rho_1(F(k_2, k_3)).$$

Hu and Wan 2020 have shown that the following map is an isomorphism of quasi-hopf, quasi-triagular Hopf algebras:

$$D^{\omega}(G) \xrightarrow{\cong} D^{\widehat{\omega}}(\widehat{G})$$
$$(a : x) \otimes \delta_{(b:y)} \mapsto \frac{1}{|A|} \sum_{\rho, \eta \in \widehat{A}} \rho(a)\eta(b)^{-1} \delta_{(xyx^{-1}:\rho)} \otimes (x : \eta)$$

Generalization to continuous groups?

What is the generalization of $\mathrm{Vec}^{\omega}(\mathrm{G})$ to continuous groups?

Related to the question:
What does Chern-Simons theory assign to a point?
But, not quite...

For G a compact, connected Lie group we have (Reshetikhin-Turaev):

$$F : \mathrm{Bord}_{\langle 3,2,1\rangle} \to \mathrm{Cat}_{\mathbb{C}}$$
$$F(S^1) \mapsto Rep({}^{k}LG)$$

$Rep({}^{k}LG)$ is the category of positive energy representations of the Loop Group LG at level $k \in H^4(BG, \mathbb{Z})$.

Idea of generalization to continuous groups

Joint work with:
J. Blanco (Ph.D. student) and K. Waldorf (Greifswald).

Extract the information of $\mathrm{Vec}^{\omega}(\mathrm{G})$ and its module categories and set it up in the 2-category of sets with $U(1)[1]$ structure.

$\mathbb{C} - \textit{categories}$	$U(1)[1]$-bundles over Sets
Pointed fusion category $\mathrm{Vec}^{\omega}(\mathrm{G})$	monoid object over G $\omega \in Z^3(G, U(1))$
Module category	Representation

2-category of geometrical $U(1)$-Gerbes

$U(1)$-Gerbes over manifolds. Objects:

$$U(1)[1] \to P \to M$$

Morita equivalence classes of extensions as groupoids

+ morphisms, + 2-morphisms...

Monoid objects, representations of these monoid objects on $U(1)$-gerbes, endomorphism category of a representation ...

Very difficult. Instead use a *homological/topological model*.

Segal-Mitchison cohomology [1970]

Coefficients:
Compactly generated and locally contractible Hausdorff topological abelian groups.

$$A \hookrightarrow EA \longrightarrow BA$$

is a short exact sequence of groups in this category.

Any group A may be resolved by contractible groups:

$$A \hookrightarrow EA \xrightarrow{\partial_0} EBA \xrightarrow{\partial_1} EB^2A \xrightarrow{\partial_2} EB^3A \xrightarrow{\partial_3} \cdots .$$

SM-cohomology of spaces

Let X be a paracompact space. Consider the complex

$$C^m(X, \underline{A}) := \mathrm{Map}(X, EB^m A), \quad \partial.$$

Segal-Mitchison cohomology of X:

$$H^*(X, \underline{A}) = H^*(C^*(X, \underline{A}), \partial).$$

Properties. Cocycles: $Z^m(X, \underline{A}) \cong \mathrm{Map}(X, B^m A)$

$$\boxed{H^m(X, \underline{A}) = \begin{cases} [X, B^m A] & m > 0 \\ \mathrm{Map}(X, A) & m = 0 \end{cases}}$$

SM cohomology of simplicial spaces

Let $X_\bullet$ be a paracompact simplicial space. Define the double complex:

$$C^{p,q}(X_\bullet, \underline{A}) := \mathrm{Map}(X_p, EB^q A)$$

and consider the total complex

$$C^*(X_\bullet, \underline{A}) := \mathrm{Tot}(C^{*,*}(X_\bullet, \underline{A})).$$

Segal-Mitchison cohomology of $X_\bullet$:

$$H^*(X_\bullet, \underline{A}) = H^*(C^*(X_\bullet, \underline{A}), \partial \pm \delta).$$

SM cohomology of topological groups

Let G be paracompact and compactly generated topological group. Let $G_\bullet$ be its simplicial space (BG).

Definition The Segal-Mitchison cohomology of G with coefficients in A is $H^*(G_\bullet, \underline{A})$.

- $H^1(G_\bullet, \underline{A}) \cong \mathrm{Hom}(G, A)$. (Segal)

- $H^2(G_\bullet, \underline{A}) \cong \mathrm{Ext}(G, A)$; A-central extensions of G. (Segal)

- $H^3(G_\bullet, \underline{A}) \cong \mathrm{Ext}(G[0], A[1])$; 2-group $A[1]$-central extensions of $G[0]$. (Schommer-Pries, Breen, Rousseau)

SM cohomology. Further properties

For G discrete or A contractible:

$$H^*(G_\bullet, \underline{A}) \cong H^*_{cont}(G, A).$$

For A discrete:

$$H^*(G_\bullet, \underline{A}) \cong H^*(BG, A).$$

Lyndon-Hochschild-Serre spectral sequence.

$$S \hookrightarrow G \to K$$

LHS spectral sequence abuts to $H^*(G_\bullet, \underline{A})$ and whose second page is:

$$E_2^{p,q} \cong H^p(K_\bullet, \underline{H^q(S_\bullet, \underline{A})}).$$

Segal-Mitchison A-gerbes

Objects: (M, α), with M paracompact and locally compact space and

$$\boxed{\alpha \in Z^2(M, \underline{A}) \cong \mathrm{Map}(M, B^2 A)}$$

Morphisms:

$$\boxed{(M, \alpha) \xrightarrow{(F,c)} (N, \beta)}$$

for $F; M \to N$ and $c \in C^1(M, \underline{A})$ with $\alpha - F^*\beta = \partial c$.

2-Morphisms:

$$\boxed{(F, c_1) \xRightarrow{e} (F, c_2)}$$

for $e \in C^0(M, \underline{A})$ such that $\partial e = c_2 - c_1$.

Multiplicative Segal-Mitchison A-gerbes

Monoidal structure:

$$(M, \alpha) \times (N, \beta) := (M \times N, \pi_M^* \alpha + \pi_N^* \beta)$$

with π_M and π_N the projections on M and N respectively.

Definition: A multiplicative SM A-gerbe $\langle M, \omega \rangle$ is a monoid object in SM A-gerbes.

Multiplicative SM A-gerbes over the monoid M
$\updownarrow$
elements $\omega \in Z^3(M_\bullet, \underline{A})$

Representations of Multiplicative SM A-gerbes

Objects: (N, β), with M acting on N and $\beta \in Z^2((N \rtimes M)_\bullet, \underline{A})$ such that

$$\boxed{d_{N \rtimes M}\beta = \pi^*\omega}$$

where $\pi : (N \rtimes M)_\bullet \to M_\bullet$ is the projection.

Morphisms:

$$\boxed{(N, \beta) \xrightarrow{(F,\gamma)} (N', \beta')}$$

for M-equivariant $F : N \to N'$ and $\gamma \in C^1((N \rtimes M)_\bullet, \underline{A})$ with $\beta - F^*\beta' = d_{N \rtimes M}\gamma$.

2-Morphisms:

$$\boxed{(F, \gamma_1) \overset{\nu}{\Longrightarrow} (F, \gamma_2)}$$

for $\nu \in C^0((N \rtimes M)_\bullet, \underline{A})$ such that $d_{N \rtimes M}\nu = \gamma_2 - \gamma_1$.

Endomorphisms of Representations

The monoidal category of endomorphisms

$$\boxed{\mathrm{End}_{\langle M,\omega\rangle}(N,\beta)}$$

could be understood as a crossed-module (2-group)

$$\boxed{C^0((N \rtimes M)_\bullet, \underline{A}) \to \mathrm{End}^0_{\langle M,\omega\rangle}(N,\beta)}$$

with

$$\pi_1 = \mathrm{Map}(N,A)^M \quad \text{and} \quad \pi_0 = \pi_0(\mathrm{End}^0_{\langle M,\omega\rangle}(N,\beta)).$$

$$H^1((N \rtimes M)_\bullet, \underline{A}) \to \pi_0(\mathrm{End}^0_{\langle M,\omega\rangle}(N,\beta)) \to \mathrm{End}_M(N,[\beta]).$$

Note: Here we need that M acts on N transitively.

Conditions for duality; $A = U(1)$

$\langle G, \omega \rangle$ multiplicative SM $U(1)$-gerbe.
S abelian and normal subgroup of G;

$$\boxed{S \longrightarrow G \longrightarrow K}$$

$$[\omega] \in \Omega(G, S) = \mathrm{Ker}(\mathrm{Ker}(H^3(G_\bullet, \underline{U(1)}) \to E_\infty^{0,3}) \to E_\infty^{1,2})$$

Choice	Reason
$S \subset G$ abelian	$\widehat{\widehat{S}} \cong S$
$[\omega]\|_S = 0$ in $H^3(S_\bullet, \underline{U(1)})$	(K, β) is a $\langle G, \omega \rangle$-rep.
$S \lhd G \to K$ normal	$\mathrm{Hom}_G(K, K) = K$
$[\omega] \mapsto 0$ in $E_\infty^{1,2}(LHS)$	Dual group surjects to K

Main result: Duality construction.

Consider the central extension of loc. contractible, compactly gen. and paracompact groups with S locally compact

$$\boxed{S \longrightarrow G \longrightarrow K}$$

Take $[\omega] \in \Omega(G, S)$ and (K, β) as before.
Theorem (Blanco-U-Waldorf-2020)

$$\boxed{U(1)[1] \longrightarrow \mathrm{End}_{\langle G,\omega \rangle}(K, \beta) \longrightarrow \widehat{G}[0]}$$

is an extension of 2-groups with

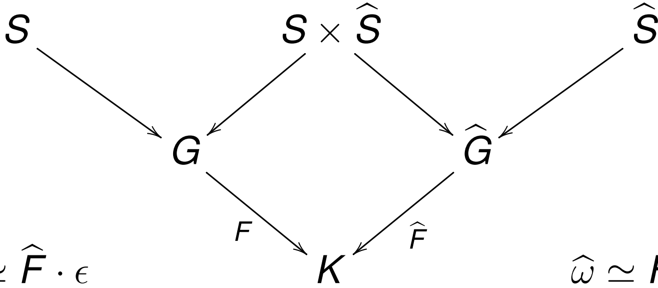
$$\boxed{\widehat{S} \longrightarrow \widehat{G} \longrightarrow K}$$

and whose extension class is $[\widehat{\alpha}_{(K,\beta)}] \in H^3(\widehat{G}_\bullet, \underline{U}(1))$.

Pontrjagin duality of multiplicative gerbes

Denote the multiplicative gerbes $\langle G, \omega \rangle$ and $\langle \widehat{G}, \widehat{\omega} \rangle$ **Pontrjagin dual**.

$$\langle G, \omega \rangle \leftrightarrow \langle \widehat{G}, \widehat{\omega} \rangle$$



$$\omega \simeq \widehat{F} \cdot \epsilon \qquad\qquad \widehat{\omega} \simeq F \cdot \epsilon$$

Examples

- S loc. compact abelian, $\boxed{\langle S, 0\rangle \leftrightarrow \langle \widehat{S}, 0\rangle}$

- $SU(2) \xrightarrow{\pi} SO(3)$, $S = \mathbb{Z}/2$, $\rho \in H^1(\mathbb{Z}/2, U(1))$

$$\boxed{\langle SU(2), 4kc_2\rangle \leftrightarrow \langle SO(3) \times \mathbb{Z}/2, kp_1 + \omega_2\rho\rangle}$$

- $\mathbb{R}^3$, $\mathcal{H} = \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$, $\alpha(\mathbf{r}, \mathbf{s}, \mathbf{t}) = e^{2\pi i r_1 s_2 t_3}$

$$\boxed{\langle \mathbb{R}^3, \alpha\rangle \leftrightarrow \langle \mathcal{H}, 0\rangle}$$

- etcetera.

“Strictification" and center

Mult. Gerbe	2-group
$\langle G, \omega \rangle$	$\mathrm{End}_{\langle G,\omega\rangle}(G, \overline{\omega})$
$\langle \widehat{G}, \widehat{\omega} \rangle$	$\mathrm{End}_{\langle G,\omega\rangle}(K, \beta)$

Let the center of the monoidal category $\mathcal{C}$ be:

$$\boxed{\mathcal{Z}(\mathcal{C}) = \mathrm{End}_{\mathcal{C}-\mathcal{C}}(\mathcal{C})}.$$

Theorem: (Blanco-U-Waldorf) Pontrjagin dual multiplicative gerbes $\langle G, \omega \rangle$ and $\langle \widehat{G}, \widehat{\omega} \rangle$ have equivalent centers

$$\boxed{\mathcal{Z}\langle G, \omega \rangle \simeq \mathcal{Z}\langle \widehat{G}, \widehat{\omega} \rangle}$$

What about representations?

The information encoded in $\mathcal{Z}\langle G, \omega \rangle$ is precisely the information required to define FHT K-theory

$$^{\tau(\omega)}K_G(G).$$

But, we still do not know how to represent $\mathcal{Z}\langle G, \omega \rangle$ in vector spaces.

If we had such construction, we expect

$$\pi_0\,(\text{“Rep"}(\mathcal{Z}\langle G, \omega \rangle)) \cong {}^{\tau(\omega)}K_G(G)$$

and therefore we would expect that Pontrjagin dual multiplicative gerbes induce isomorphic Verlinde algebras...

Conclusions

We have defined (cohomological) Segal-Michison gerbes.

In this category we are able to define Pontrjagin dual multiplicative gerbes (this duality is also known as **Electric-magnetic duality**).

We show that Pontrjagin dual multiplicative gerbes have equivalent centers.

We have generalized the Twisted Drinfeld Double to topological groups.

This talk is based on the articles

- Blanco, U., Waldorf. *Pontrjagin duality on multiplicative gerbes.* arxiv:2012.05056
- Maya, Castaño, U. *Classification of pointed fusion categories of dimension p^3 up to weak Morita equivalence.* J. Algebra Appl. (2021).
- Muñoz, U. *Classification of pointed fusion categories of dimension 8 up to weak Morita equivalence.* Comm. Algebra (2018).
- U. *On the classification of pointed fusion categories up to weak Morita equivalence.* Pacific J. Math. (2017).

THANKS!